Artifacts and Errors in Cross-Spectrum Phase Noise Measurements

— Invited lecture —

Yannick Gruson[∃ Θ], Adrian Rus[◊], Alexander Roth[□] and Enrico Rubiola[∃ ∇ Θ ∀]

***Abstract*—Inserting an attenuator between the oscillator under test and the phase noise analyzer, one expects that the white phase noise increases monotonically with the attenuation. By contrast, we observe that with some oscillators the white noise has sharp minimum for a given value of the attenuation, which clearly indicates problem. With other oscillators, it increases monotonically with the attenuation, but the values are not consistent with the thermal energy introduced by the attenuator. In both cases artifacts are present, which takes the form of a sharp notch in the spectrum, occurring where the white FM noise crosses the white PM noise. Such anomalous behavior is the tip of the iceberg, and reveals a common misconception in the cross spectrum instruments and in their use. We provide analytical theory, experimental results, and tips to mitigate the problem.**

The lecture and this extended abstract are based on the results published in Y. Gruson et al. *Metrologia* 57(5) p. 055010 1-12, DOI 10.1088/1681-7575/ab8d7b [1]. The latter is the reference article for the reader interested in deeper understanding.

I. Introduction

After the pioneering article [2] and the early application with commercial FFT analyzers [3], the cross spectrum method has been used extensively for the measurement of phase noise. The major benefit of the cross spectrum method is the rejection of the instrument background noise by correlating and averaging the output of two equal channels. Nowadays most commercial instruments are based on this method. It has been demonstrated that the residual background is limited by the thermal uniformity, rather than the absolute temperature of the instrument [4]. Later, it has been pointed out that the same applies to the measurement of oscillators [5], [6], [7]. In [8], we suggested the that the cross spectrum can be negative in some not-so-rare unfortunate circumstances.

II. Experiment and Results

We run the experiment shown on Fig. 1 sweeping the attenuation up to 27 dB in 3 dB steps. The oscillator under test is (A) a Wenzel 501-04623E or (B) a Wenzel 501-25900B "Golden Citrine." Both are 100 MHz OCXOs intended for lowest noise applications. The oscillator (A) is an old design, likely from the 1990s, while (B) is one of the latest. The phase noise analyzer is a Rohde Schwarz FSWP-26 with full low-noise options. Despite the rather high attenuation, the signal is still in the input range of the analyzer.

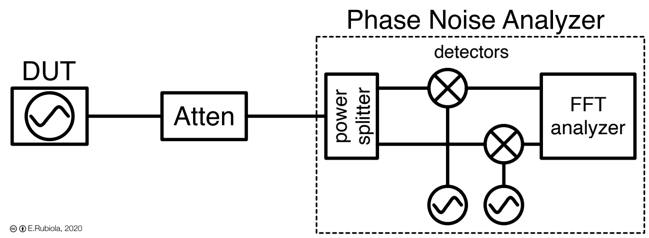

Fig. 1. Experiment. Two 100-MHz OCXOs are tested, (A) Wenzel 501-04623E, ouput power $P = 9.8$ dBm, and (B) Wenzel 501-25900B "Golden Citrine," $P = 18.5$ dBm. The analyzer is a FSWP-26. The original picture is © E. Rubiola 2020, released under Creative Common CC-BY 4.0 license.

An excerpt of the results is shown on Fig. 2, where we identify the white FM noise $S_\varphi(f) = \mathsf{b}_{-2}/f^2$ and the white PM noise $S_\varphi(f) = \mathsf{b}_0$. Oddly, 15 dB attenuation in Fig. 2-top results in b_0 10 dB smaller than with no attenuation. At 27 dB attenuation, b_0 gets higher, albeit it seems unrelated to the previous values. A sharp notch shows up at $f = 1$ kHz, where the white FM crosses the white PM.

In Fig. 2-bottom, b_0 increases monotonically when the attenuation increases, but the values are inconsistent with the physics of the attenuator. In fact, the expected white noise is $N = kT_iA^2 + kT_a(1 - A^2)$ [W/Hz], where k is the Boltzmann constant, T_i is the equivalent noise temperature at the attenuator input (oscillator output), T_a is the temperature of the attenuator, and A is the 'gain' of the attenuator, $A^2 < 1$. In turn, the white PM noise is $\mathsf{b}_0 = kT_i/P$ at the attenuator input, and $\mathsf{b}_0 = N/P_o$ at the attenuator output. Additionally, the sharp notch is present where the white FM crosses the white PM.

The results are reproducible, to the extent that

1) we got similar pattern with other low-noise OCXOs,
2) the same behavior is observed with a Keysight E5052B phase noise analyzer,
3) three set of experiments were carried on separately, in Besançon, Bucharest and München with different specimens.

Here, we report only on the first set. Interestingly, the two analyzers are quite different from one another. The FSWP is based on Software Defined Radio analysis of the signal down-converted to an appropriate IF, and has a directional

∃ FEMTO-ST Institute, Dept. of Time and Frequency, Université de Bourgogne and Franche-Compté (UBFC), and CNRS.
Θ Observatory of Besançon, France.
◊ Famagusta, Bucharest, Romania. Call sign YO3HHZ.
□ Rohde & Schwarz, München, Germany.
∇ Istituto Nazionale di Ricerca Metrologica INRiM, Torino, Italy.
∀ Invited, speaker, and reference author, e-mail: rubiola@femto-st.fr. Home page http://rubiola.org.

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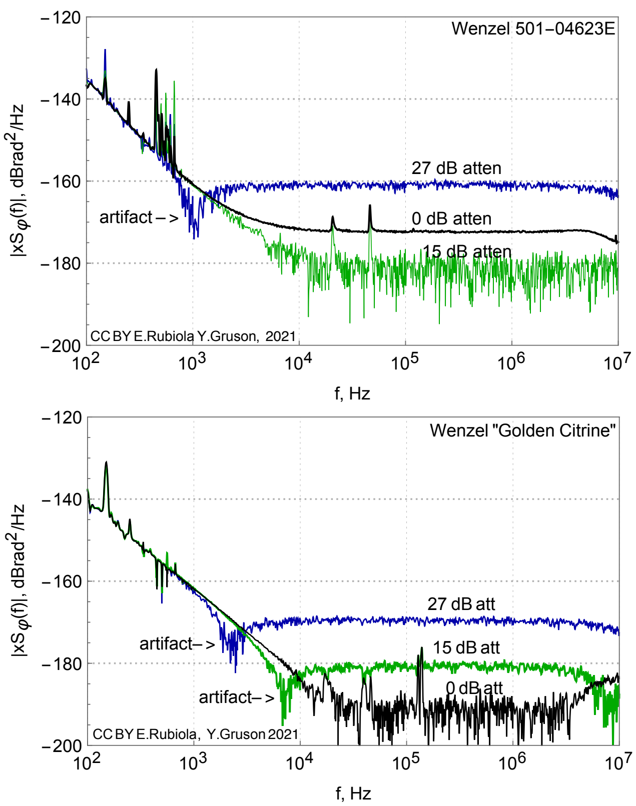

Fig. 2. Results. Top: oscillator (A), bottom: oscillator (B). The original pictures are © E. Rubiola and Y. Gruson 2020, released under Creative Common CC-BY 4.0 license.

coupler (3 dB attenuation, plus losses) as the input power splitter. Conversely, the E5052B is based on phase-to-voltage conversion with saturated mixers, and has a Y resistive power splitter (6 dB attenuation, plus losses). However, the large-signal calibration (white FM) is consistent within 0.1–0.2 dB.

III. What Happens

The following phenomena relate to our observations

1) The thermal energy in the input power splitter contributes with its thermal energy kT_s, which has negative sign in the cross spectrum [5], [6], [8]. This introduces a negative bias error.
2) Crosstalk between the two channels is inevitable in the real world, due to finite isolation. In our case, we observed an anti-correlated noise, which we can represent as kT_c, with $T_c = -122$ K. This is a characteristic parameter of the instrument in our experimental conditions because a single value describes well the results of Fig. 2, for both oscillators and for all values of A. The crosstalk introduces a further bias error, which is negative in the experiments described.
3) The instruments use the absolute value as the estimator, that is, $\widehat{S_\varphi(f)} = \langle|S_{yx}(f)|\rangle_m$, averaged on m acquisitions. We have shown in [9] that the *signal* (the DUT noise) goes only in $\Re\{S_{yx}(f)\}$, while $\Im\{S_{yx}(f)\}$ contains only the background noise of the instrument. However, the combined effect of T_s and T_c is such that $\Re\{S_{yx}(f)\}$ can be negative. This occurs with attenuation beyond 15 dB in Fig. 2-top, and for all the plots of Fig. 2-bottom. Thus, flipping the sign is a nonsense. To prove our ideas, we hacked a FSWP at the Rohde Schwarz R&D facility, extracting $\langle\Re\{S_{yx}(f)\}\rangle_m$ and $\langle\Im\{S_{yx}(f)\}\rangle_m$ separately, and using $\widehat{S_\varphi(f)} = \langle\Re\{S_{yx}(f)\}\rangle_m$ as the estimator.

Measuring an oscillator of the same type of (B), we got a negative value of the white PM noise term b_0.

IV. Final Remarks

The pathologies reported occur only in rather extreme measurements, and should not alarm the general practitioner. However, when the PM noise is smaller than 10–15 dB above kT/P, i.e., the thermal energy divided by the carrier power, errors are around the corner.

The foundations of this article are published in the open-access article [1]. A companion slideshow is also available on `http://rubiola.org`.

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